Chapter 8 - Day 2

<u>Ex</u>: Estimate the area under the graph of $y = \frac{1}{x}$ for x between 1 and 31 by subdividing into 30 equal subintervals and using the left endpoint as a sample point.

width of subinterval $\frac{31-1}{30} = \frac{30}{30} = 1$

$$\text{Area} = 1 \cdot f(1) + 1 \cdot f(2) + 1 \cdot f(3) + \ldots + 1 \cdot f(30)$$

$$= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{30}$$

$$= \boxed{3.995}$$

Sigma Notation

$$a_1 + a_2 + a_3 + \ldots + a_n = \sum_{k=1}^{n} a_k$$

k is called the index or counter.

Ex: find $\sum_{k=2}^{5} k^2$

$$\sum_{k=2}^{5} k^2 = 2^2 + 3^2 + 4^2 + 5^2$$

$$= 4 + 9 + 16 + 25$$

$$= \boxed{54}$$

__Ex__: $\displaystyle\sum_{k=1}^{5}(2k-1)$

$= (2\cdot1-1)+(2\cdot2-1)+(2\cdot3-1)+(2\cdot4-1)$
$+(2\cdot5-1)$

$= 1+3+5+7+9 = \boxed{25}$

__Ex__: $\displaystyle\sum_{k=1}^{5}(3k^2+k)$

$= (3\cdot1^2+1)+(3\cdot2^2+2)+(3\cdot3^2+3)+(3\cdot4^2+4)$
$+(3\cdot5^2+5)$

$= 4+14+30+52+80$

$= \boxed{180}$

Ex: $\sum_{k=1}^{112} 75$

$$= \underbrace{75 + 75 + 75 + \dots + 75}_{112 \text{ times}}$$

$$= 112 \cdot 75$$

$$= \boxed{8400}$$

Ex: $\sum_{k=15}^{273} 23$

$$= \underbrace{23 + 23 + 23 + \dots + 23}_{273-14 \text{ times}}$$

$$= 259 \cdot 23$$

$$= \boxed{5957}$$

The technical term for "subdivide an interval" is "<u>partition an interval</u>"

For example, when we subdivided $[0,2]$ into 4 subintervals, we say it has the partition $\{0, \frac{1}{2}, 1, \frac{3}{2}, 2\}$

with subintervals $[0,\frac{1}{2}], [\frac{1}{2},1], [1,\frac{3}{2}], [\frac{3}{2},2]$

The Definite Integral

let $f(x)$ be a function defined on $[a,b]$.
Partition the interval $[a,b]$ into n Subintervals of lengths $\Delta x_1, \dots, \Delta x_n$ respectively. for $k=1,\dots,n$ Pick a representative point p_k in the k^{th} subinterval.

The definite integral of the function f from a to b is defined as

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(p_k)\cdot\Delta x_k$$

$$= \lim_{n\to\infty} \left(f(p_1)\cdot\Delta x_1 + f(p_2)\cdot\Delta x_2 + \dots + f(p_n)\cdot\Delta x_n\right)$$

and is denoted

$$\int_a^b f(x)\,dx$$

upper limit of integration

integral sign

$$\int_a^b f(x)\,dx$$

integrand

differential of x

lower limit of integration

"Riemann Sum"

<u>Note</u>: x is a dummy variable, that is

$$\int_a^b x^2\,dx = \int_a^b t^2\,dt$$

they represent the same number

but in Ch 3, we learned limits don't always exist....

<u>Theorem</u>: let $f(x)$ be a continuous function on the interval $[a,b]$ then $\int_a^b f(x)\,dx$ exists.